Grade 7 Solving simple linear equations

7.PR.6	
Model and solve problems that can be represented by one-step linear equations of the form x + a = b, concretely, pictorially, and symbolically, where a and b are integers.	1. Represent a problem with a linear equation and solve the equation using concrete models. 2. Draw a visual representation of the steps required to solve a linear equation. 3. Solve a problem using a linear equation. 4. Verify the solution to a linear equation using concrete materials and diagrams. 5. Substitute a possible solution for the variable in a linear equation to verify the equality.

7.PR.7	
Model and solve problems that can be represented by linear equations of the form: • ax + b = c • ax = b • x/a = b, a not equal 0 concretely, pictorially, and symbolically, where a, b, and c are whole numbers.	6. Model a problem with a linear equation and solve the equation using concrete models. 7. Draw a visual representation of the steps used to solve a linear equation. 8. Solve a problem using a linear equation and record the process. 9. Verify the solution to a linear equation using concrete materials and diagrams. 10. Substitute a possible solution for the variable in a linear equation to verify the equality.

Clarification of the outcome:

- ✦ The two outcomes are strongly connected. They should be combined for that reason.
- ✦ The equations are called linear because if you graph them, the graph is a line.
- ✦ For the equation types, ax = b, x/a = b, and ax + b = c, 'a' must always be a positive whole number. Multiplication or division by a negative integer is a grade 8 outcome. To avoid this, ensure the equations you use in the activities do not involve multiplication or division with negative integers.

Required close-to-at-hand prior knowledge:

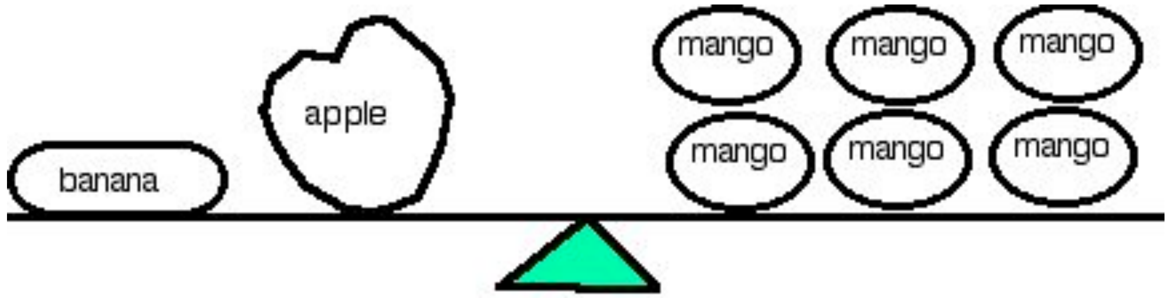
- ✤ Able to evaluate expressions (7.PR.5).
- ✤ Mental arithmetic skills, especially the basic facts of arithmetic.
- ✤ Proficient with integer addition and subtraction.

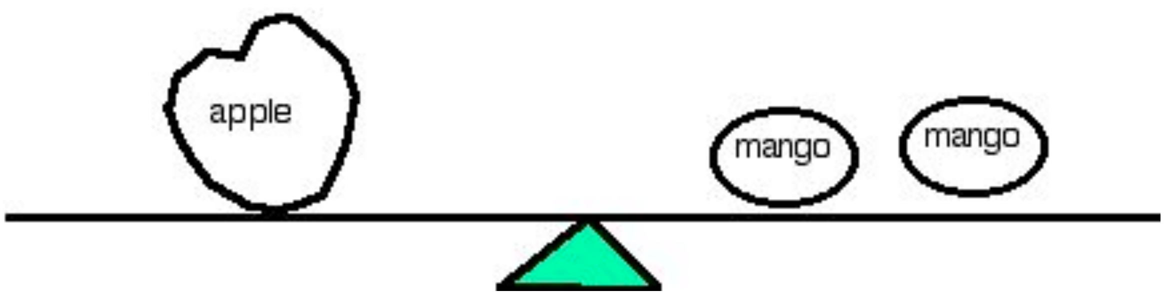
SET SCENE stage

The problem task to present to students:

Present the following balance beam barter problem.

J*ohn is visiting a village where barter is used to obtain what you need. John is hungry for a banana. All he has are some mangoes in his backpack. The banana seller has a sign that shows bartering rules for mangoes, apples, and bananas (see diagram). How many mangoes will John need to trade for one banana?*





Comments:

The main purpose of the task is to provide an experience with a balance beam model for equality and an acceptable student-friendly reason for solving equations.

Note:

If desired, Refer to 'The **Land of Pome**'. The story A fair day to trade for two additional balance beam barter problems.

DEVELOP stage

Activity 1: Revisits SET SCENE and addresses indicators, 2, 4, 5, 6, 7, and 9.

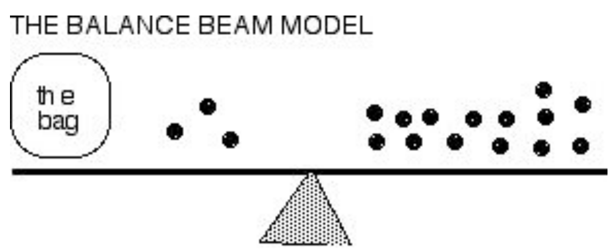
- Ask some students to explain their solutions to the banana problem.
- Present them with a more difficult problem (see following example). Discuss ways to solve it BUT do not solve it. Suggest that a solution method that involves manipulating variables might be a more powerful way to tackle such problems. Suggest that the goal is to learn solution methods for solving such problems.

Three fish tanks (small, medium, and large) hold four kinds of fish (angel, guppy, blue, and zebra). There are 32 fish altogether in the tanks. In all, there are 6 angel fish and 12 guppies. The small tank has 8 guppies and 4 blue fish in it. The medium tank has 2 angel fish and 4 blue fish in it. The large tank has 6 zebra fish and other kinds of fish. How many fish in all are in the large tank? How many of each kind are in the large tank?

Activity 2: Addresses achievement indicators 1 through 10.

- Ask students to solve the following problem by thinking in terms of a balance beam model (see diagram).

Johnny, a friend of mine, gives me a bag with some goodies in it. He does not tell me how many goodies are in the bag. I put the bag in my pocket. Then his brother gives me 3 goodies as well to put in my pocket. Johnny grins and says to me "Now you have 14 goodies in your pocket. How many goodies are in the bag?



A FORMAL SYMBOLIC METHOD

$x + 3 = 14$
$x + 3 - 3 = 14 - 3$
$x + 0 = 11$
$x = 11$

CHECKING BY SUBSTITUTING

$x + 3 = 14$
$11 + 3 = 14$
$14 = 14$

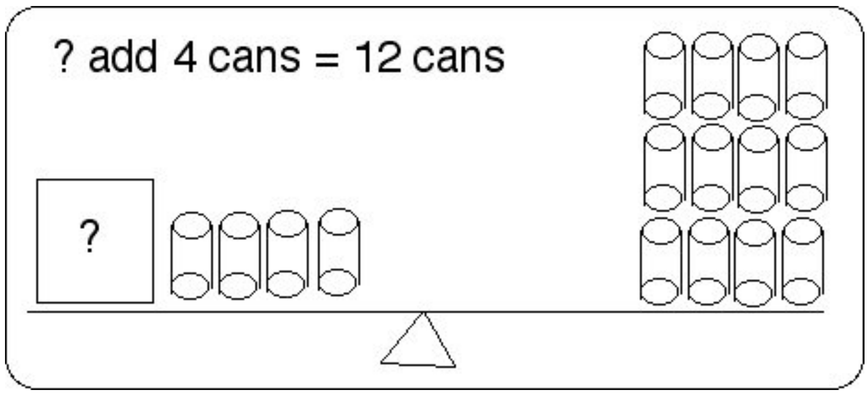
- After students have solved the problem by removing goodies, discuss how the problem can be written in a way that involves a variable. Ensure that they understand that the equation, ‘x + 3 = 14, can be used to represent the problem. Discuss an algebraic approach to solving the problem (see diagram). Ensure they understand the linkage between the algebraic approach and the balance beam approach that involves removing 3 goodies from each side. Have students check the solution by substituting the numerical value for the variable (‘x’, in this example).

Activity 3: Addresses achievement indicators 1 through 10.

- ✦ Ask students to represent the following problem as an equation. [Expect: $x + 4 = 12$]

  A very light box along with four cans balances twelve cans. How many cans are in the box?

- ✦ Have them solve the problem by representing the equation in balance beam form (see diagram) and manipulating concrete objects (e.g. removing cans). Discuss their solutions.



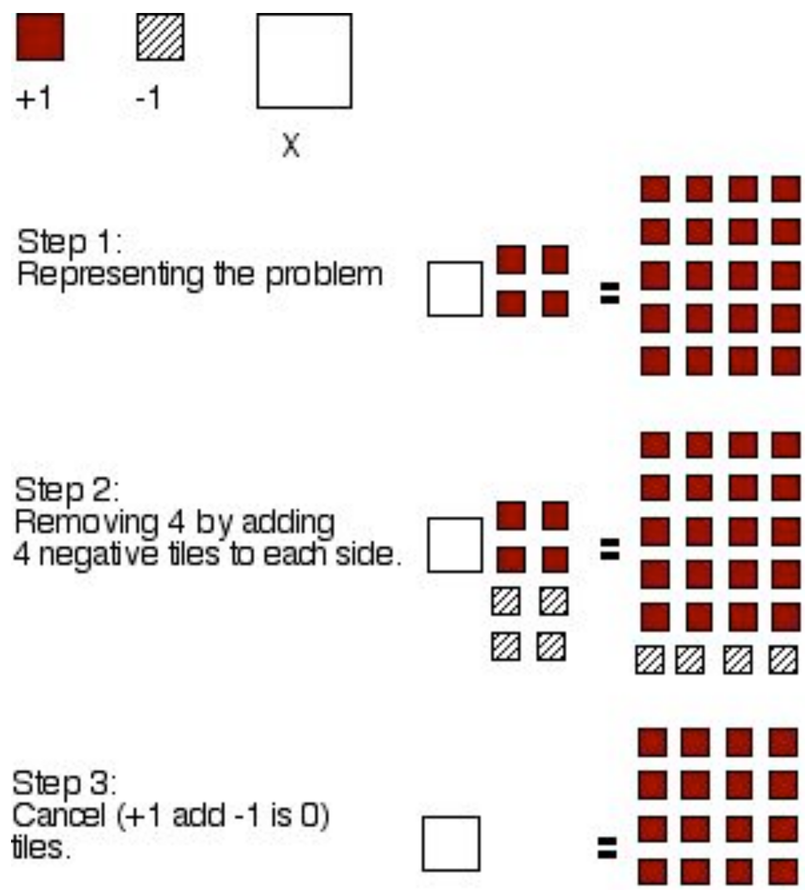
- ✦ Have students solve the problem algebraically (as developed in activity #2). Discuss their solutions.

Activity 4: Addresses achievement indicators 1 through 10.

- ✦ Ask students to represent the following problem as an equation.

  A very light box along with 4 pennies balances twenty pennies. How many pennies are in the box

- ✦ Have them solve the problem using algebra tiles (see diagram).



- ✦ Have them solve the problem algebraically. [x + 4 = 20; x + 4 - 4 = 20 - 4; x = 16] Discuss the connection between the algebra tiles method and the algebraic method.

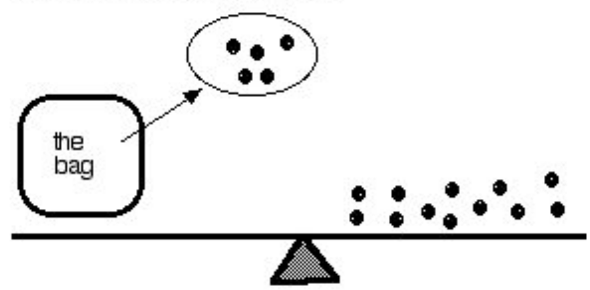
Activity 5: Addresses achievement indicators 1 through 10.

- Ask students to solve the following problem by thinking in terms of a balance beam model (see diagram).

  I *have a bag with some marbles in it in my pocket. I remove five marbles from the bag. Now I have 12 marbles in the bag. How many marbles did I have in the bag to begin with?*

- After students have solved the problem by adding marbles to undo the removing of marbles, discuss how the problem can be written in a way that involves a variable. Ensure that they understand that the equation, ‘x - 5 = 12’, can be used to represent the problem. Discuss an algebraic approach to solving the problem (see diagram). Ensure they understand the linkage between the algebraic approach and the balance beam approach that involves adding 5 marbles to each side. Have students check the solution by substituting the numerical value for the variable (‘x’, in this example).

A BALANCE BEAM MODEL



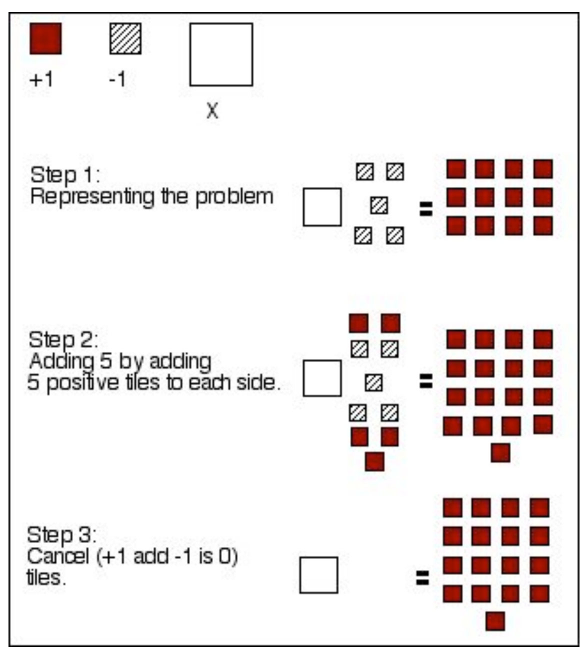
A FORMAL SYMBOLIC METHOD

$x - 5 = 12$
$x - 5 + 5 = 12 + 5$
$x + 0 = 17$
$x = 17$

CHECKING BY SUBSTITUTING

$x - 5 = 12$
$17 - 5 = 12$
$12 = 12$

- Ask students to solve the same problem using algebra tiles (see diagram). Ensure they understand the linkage between the algebraic approach and the algebra tile approach.



Activity 6: Addresses achievement indicators 1 through 10, and practice.

- Provide students with four equations, two of the form 'x + b = c' (e.g.: x + 7 = 13) and two of the form 'x - b = c' (e.g.: x - 3 = 10). Have them solve the equations algebraically and check their solutions. Discuss solutions. Provide algebra tiles as needed.
- Provide students with four equations, two of the form 'x + b = c' (e.g.: x + 7 = -13) and two of the form 'x - b = c' (e.g.: x - 3 = -10). Have them solve the equations algebraically. Discuss solutions. Provide algebra tiles as needed.

Activity 7: Assessment of teaching (part 1).

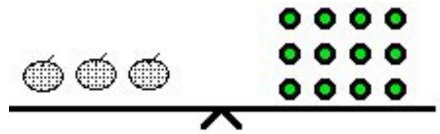
- Because this is a long lesson, it might be wise to assess your teaching at this point (solving equations of the form: x + b = c). If students have issues solving this type of equation, they cannot hope to understand the more complex type yet to come.
- Present students with two equations: (1) x - 3 = 4 and (2) x + 5 = -1. Ask students to solve the problems using algebra tiles (have them draw the steps involved) and the algebraic method.

Activity 8: Addresses achievement indicators 1 through 10.

- Ask students to represent the following problem as an equation.

  Three apples balance 12 grapes. How many grapes will balance one apple?

- Have them solve the problem by representing the equation in balance beam form (see diagram) and manipulating concrete objects (e.g. grouping grapes). Discuss their solutions.



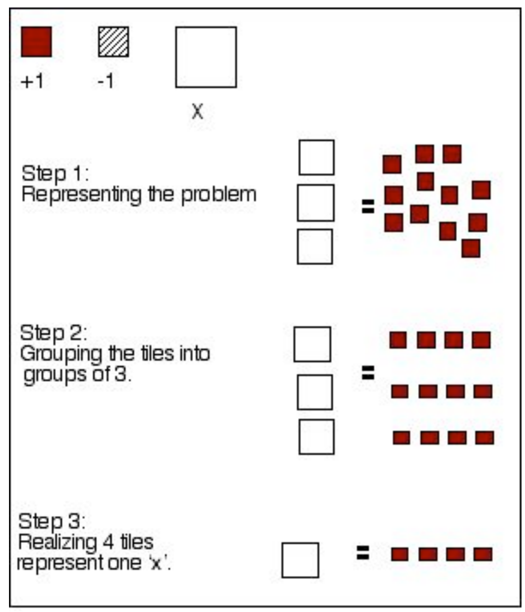
$$4x = 12$$

$$4x/4 = 12/4$$

$$x = 3$$

- Have them solve the problem by using algebra tiles (see diagram). Discuss their solutions.

- Discuss an algebraic approach to solving the problem (see diagram above). Ensure they understand the linkage between the algebraic approach and the balance beam and algebra tiles approach. Have students check the solution by substituting the numerical value for the variable (‘x’, in this example).



Activity 9: Addresses achievement indicators 1 through 10, and practice.

- Provide students with four equations, of the form 'ax = b', where 'a' and 'b' are whole numbers (e.g. 2x = 10).

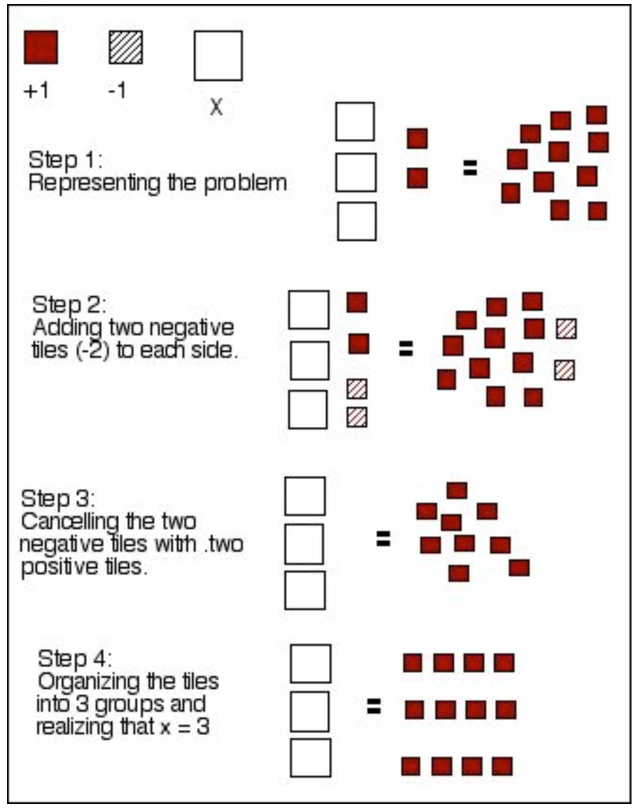
- Have them solve the equations using algebra tiles and the algebraic method. Discuss solutions.

Activity 10: Addresses achievement indicators 1 through 10.

- Ask students to represent the following problem as an equation.

  Three stars and two squares balance 11 squares. How many squares will balance one star?

- Have them solve the problem by using algebra tiles (see diagram) Discuss their solutions.



- Discuss an algebraic approach to solving the problem (see example). Ensure they understand the linkage between the algebraic approach and the algebra tiles approach. Have students check the solution by substituting the numerical value for the variable ('x' in this example).

$$3x + 2 = 11$$

$$3x + 2 - 2 = 11 - 2$$

$$3x = 9$$

$$3x/3 = 9/3$$

$$x = 3$$

Activity 11: Addresses achievement indicators 1 through 10, and practice.

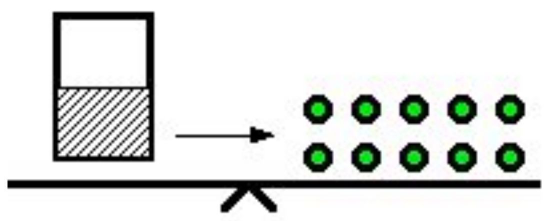
- Provide students with four equations of the form 'ax + b = c' (e.g. 2x + 1 = 7).
- Have them solve the equations using algebra tiles and the algebraic method. Discuss solutions.

Activity 12: Addresses achievement indicators 1 through 10.

- Ask students to represent the following problem as an equation.

  Half of a box balances 5 nails. How many nails will balance a whole box?

- Have them solve the problem by using a balance beam (see diagram). [They will need to realize that if 1/2 a box balances 5 nails, if we fill up the box, this is like doubling the number of nails to maintain balance.] Discuss their solutions.



$$x/2 = 10$$

$$2(x/2) = 2(10)$$

$$x = 20$$

- Discuss an algebraic approach to solving the problem (see diagram). Ensure they understand the linkage between the algebraic approach and the balance beam approach. Have students check the solution by substituting the numerical value for the variable (‘x’ in this example).

Note:

Using algebra tiles for equations of the form 'x/a = b' does not work well. The tiles are not appropriate for representing this type of equation. A balance beam approach is more appropriate.

Activity 13: Addresses achievement indicators 1 through 10, and practice.

- Provide students with four equations of the form 'x/a = b', where 'a' and 'b' are whole numbers (e.g. x/5 = 3).
- Have them solve the equations using algebra tiles and the algebraic method. Discuss solutions.

Activity 14: Addresses achievement indicators 1 through 10, and practice.

- Provide students with problems that can be represented with equations. Have students represent each problem with an equation, solving it algebraically. [Provide algebra tiles as needed.] A sample problem follows.

  Joe has his CDs stored in two boxes on a shelf with the same number of CDs in each box. Lying next to the boxes are 3 CDs that didn't fit into the boxes. Joe remembers that he had 51 CDs but he doesn't know how many CDs he put in each box. He needs the information for his friends. How many CDs are in each box?

Activity 15: Revisits SET SCENE & addresses indicators 1 through 10, and practice..

- Revisit the SET SCENE task. Organize students into groups of 2. Ask each group to create a similar problem that can be represented with one of the types of equations they have just learned to solve algebraically. Ask the groups to solve their equations.
- Ask selected groups to present their problem, equation, and solution. Discuss the results.

Activity 16: Assessment of teaching (part 2).

- Provide students with one equation of the form 'x + a = b', one equation of the form 'x - a = b', one equation of the form 'ax = b', one equation of the form 'ax + b = c', and one equation of the form 'x/a = b'. Ask them to solve the equations algebraically. Ask them to resolve the equation of the form 'ax + b = c' using algebra tiles and to explain the thinking involved.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

Solve the following equations using the algebraic method. Explain the thinking involved and check your answer by substituting the solution for ‘x’

a) $2x = 18$

b) $x/3 = 5$

c) $2x - 3 = 7$

d) $x + 6 = 14$

e) $x - 2 = 8$

f) $4x + 5 = 13$

MAINTAIN stage

Mini-task example

Every so often:

- Present an equation of the ‘ax + b = c’and the ‘x/a = b’ type for students to solve. Encourage them to solve it using the algebraic method.

Rich Learning Task

Providing students with problems that are can be solved using equation solving techniques. Whenever feasible, make use of contexts that arise from science or social studies to frame problems. In this way, another subject area is integrated with equation solving. Here is an example.

The total vitamin C content of an orange and a strawberry is 250 mg. The strawberry contains 147 mg of vitamin C. How much vitamin C does the orange contain?